

Fuzzy Numbers In Decision-Making Under Unpredictable Human Learning Behavior

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Abstract

Fuzzy numbers are vital to decision-making, particularly when human learning behavior is unexpected and hard to understand. Cognitive ability, emotional emotions, social interactions, and prior experiences all affect human learning. Thus, decision-makers struggle to forecast how people will react to new information, training, or decision-making situations. Because human behavior is ambiguous and variable, traditional decision-making frameworks that employ exact facts and deterministic results typically fail. Fuzzy numbers, an extension of fuzzy logic, enable inaccurate or ambiguous facts to be represented, making decision-making more flexible and realistic. Fuzzy numbers let decision-makers attribute degrees of membership to multiple circumstances instead of using fixed values. This makes them valuable in contexts where learning patterns might alter over time or between people and impact choices. Employees' reactions to new technologies or procedures in the workplace might vary. Some workers learn rapidly, while others struggle, and their learning results are hard to anticipate. Fuzzy numbers help decision-makers to represent uncertainty and explore several learning outcomes. This allows more robust and adaptable solutions to address human learning's unpredictability.

Keywords: decision-making, Fuzzy numbers, human learning behavior, Employees, fuzzy logic.

Introduction

Within the realm of research on fuzzy set theory, fuzzy numbers are a significant topic of discussion. Due to the fact that fuzzy numbers are suitable for expressing unknown values, they have found widespread usage in a variety of applications. Additionally, several techniques for manipulating fuzzy numbers have been created along with them. A variety of application domains, including decision making, have made use of the findings of research that were conducted on the ranking of fuzzy numbers. It is necessary to assess and compare one fuzzy number to the others in order to rank fuzzy numbers; nevertheless, this may not be a simple task to do initially. Due to the fact that fuzzy numbers are represented by probability distributions, it is possible for them to overlap with one another. As a result, it is quite challenging to ascertain with absolute certainty if one fuzzy number is greater or lesser than another. A wide variety of approaches to ranking fuzzy numbers have been suggested, including the use of fuzzy relations and the representation of fuzzy numbers with numerical values. On the other hand, the majority of the approaches

that have been developed in the past do not take into account the general possibility distribution of a fuzzy number, nor do they use a fixed perspective while assessing them. Fuzzy numbers, as was noted before, are a representation of uncertain values; hence, the total possibility distribution is significant and must be taken into consideration when they are evaluated. In addition, the assessments of fuzzy numbers are susceptible to being influenced by an evaluation perspective, such as the preference or interest of the user. In prior studies, a few different ways for shifting perspectives were proposed; nevertheless, the majority of these approaches were straightforward. Optimistic and pessimistic perspectives were the only ones that were taken into consideration in the majority of the earlier study, and the data from both perspectives were linearly integrated. Typically, users were the ones who supplied the combining rate, which was represented by a real number in the range $[0,1]$.

Fuzzy Number

The expression of Fuzzy number in a fuzzy set defines a fuzzy interval in the real number. This interval becomes a fuzzy set as its boundary is ambiguous. The representation of a fuzzy interval is done by two end points a_1 and a_3 and peak point a_2 as $[a_1, a_2, a_3]$. The α -cut operation can be also applied to the fuzzy number. If α -cut interval for fuzzy number is denoted as A as A_α , the obtained interval A_α is defined as $A_\alpha = [a_1(\alpha), a_3(\alpha)]$.

Since our idea of the proper value of some measurements might fluctuate depending on the degree of confidence we have in the information, a fuzzy number may be included in a practical description. This is because of the fact that our knowledge of the correct value of certain measurements can shift. As a result of this, the possibility of a fuzzy number appearing in a description is not completely out of the question.

In the beginning, the idea of fuzzy numbers and the arithmetic operations associated with them were given and examined. Additionally, the concept of a fuzzy number was provided as a fuzzy subset of the real line. When it comes to intuitionistic fuzzy numbers, the term An introduction was made to the triangular intuitionistic fuzzy number, and it was used for the purpose of reliability assessment. Additionally, a trapezoidal intuitionistic fuzzy number (TrIFN) was presented, along with the arithmetic operations of the TrIFN, which were constructed using the α -cut approach. I have provided an explanation of the Trapezoidal Intuitionistic fuzzy number (TrIFN) as well as their operational principles. introduced the concept of symmetric trapezoidal intuitionistic fuzzy numbers (STrIFNs) and discussed the desired characteristics and arithmetic operations associated with these numbers based on α -cut.

Fuzzy Decision-Making

Environments that are conducive to fuzzy decision-making provide approaches that may be used to action towards unpredictability, which manifests itself as imprecision and vagueness. Two different types of programmes, namely fuzzy flexible programming and fuzzy probabilistic programming, may be used to investigate fuzzy decision-making techniques. To illustrate the tractability of the target values of objective functions and the pliability of restraints, fuzzy flexible programming models include adaptable goals and soft restraints. These models are characterised by their ability to describe them. To put it another way, fuzzy goals and sets are a way of describing the ambiguity that is connected with the decision maker's aims and limits in the context of flexible programming. A significant limitation of fuzzy

flexible programming is that it is unable to take into account the impregnability of the constants of goal functions and restrictions. This is the most significant disadvantage of this method. It is possible to cope with uncertain elements in objective functions and limits via the use of fuzzy probabilistic programming, which makes it believable. Fuzzy probabilistic programming provides an effective design in situations when it is difficult to precisely describe the value of model variables. This is accomplished by the utilisation of probability distributions to represent fuzzy restrictions that are known on controlled accessible facts, in addition to the familiarity and habits of decision-makers.

Human Learning Behavior

Throughout the stages of human life, human conduct refers to the potential and demonstrated ability for active participation in physical, mental, and social activities. Humans, like other animal species, have a normal life cycle that consists of consecutive periods of development, each of which is characterised by a specific set of physical, physiological, and behavioural characteristics. This is the case for all animal species. These stages include prenatal life, childhood, adolescence, and adulthood (including old age). Prenatal life is the first stage of life. The subject of study known as human development, often known as developmental psychology, is an area of research that attempts to describe and explain the changes that occur in human cognitive, emotional, and behavioural capacities and functioning over the whole of a person's life, beginning with the fetus and continuing into old age.

Objectives

1. To study the determination of the many human elements that have an impact on the process of decision-making.
2. To study the consideration of human aspects within previously established decision-making procedures.

FUZZY LOGIC AND FUZZY DECISION-MAKING SYSTEM

Fuzzy logic, which is a subset of soft computing and follows an approach that is inspired by nature, serves as the basis for this article. In order to provide a framework for this study endeavor, this section will provide a short discussion of the concept of fuzzy set theory. When dealing with increasingly complicated systems, fuzzy logic is a modeling method that proves to be more effective. Through the use of fuzzy logic (FL), it is feasible to approximate the interpolation between the input and output possibilities. To handle circumstances in which the absence of well-defined class membership requirements, as opposed to the presence of random variables, is the source of imprecision, FL provides a conceptual framework that aims to develop a natural manner of treating the problem. This approach to issue resolution was established by Lotfi Zadeh in 1965 with the intention of addressing circumstances that were confusing or imprecise. Simulation of human knowledge and thinking that lacks clear bounds is accomplished via the use of FL. The four fundamental ideas of fuzzy logic are fuzzy sets, linguistic variables possibility distributions (membership functions), fuzzy if-then rules, and fuzzy if-then functions. This is despite the fact that fuzzy logic encompasses a wide range of theories and approaches. One way to statistically characterize the values of a linguistic variable is via the use of a fuzzy set. When

a linguistic variable is assigned to a fuzzy set, it is possible to put restrictions on the value of the variable. These constraints are referred to as possibility distributions or membership functions. The fuzzy if-then rules, which are knowledge representation schemes, are used to express a functional mapping that exists between antecedents and consequents.

FUZZY OPTIMIZATION

It is a technique for allocating with the imprecision and vagueness in ambiguous limitations, which are represented by fuzzy elements, and membership to an accurate set is uncertain. Fuzzy optimization is a way for allocating with these ambiguous restrictions. One of the most significant advantages of fuzzy optimization is that it enables us to deal with a wide variety of indecisions in the structure of the problem. These indecisions include the changeability in the decision maker's objective level with regard to the objective(s), the changeability in the range of constants of objective function(s) and restraints, and the ambiguity in the contentment level of restraints. This is in contrast to the robust optimization method, which only deals with indecisions in parameters.

As a result of the need of assigning with material that arises from computational observation, which may be described as being inexact, unclear, imprecise, ambiguous, or without distinct limits, Zadeh (1965) was the first person to construct a mathematical approach and give it the term Fuzzy Set Theory (FST). This idea has been used in a wide variety of engineering and scientific domains ever since it was provided to the scientific community. There are several areas of operations research that have benefited from the use of FST, including decision theory, project management, inventory control, scheduling, supply chain management, queuing theory, maintenance and dependability, and quality control, to name a few examples. FST has been used with modeling and optimization approaches such as linear and nonlinear programming, dynamic programming, integer programming, simulation, and decision-making based on multiple criteria and multiple objectives. When there is a lack of confirmation that can be obtained or a lack of inevitability in recommendation, optimization and modeling techniques, when coupled with FST, give the appropriate framework to identify and handle natural indecisions. This is done in order to deal with the ambiguous nature of flexible supply chains.

MULTI-CRITERIA DECISION-MAKING PROBLEM SOLVING

Before going on to the details of the problem formats themselves, this section provides an overview of the essential concepts that underpin multi-criteria decision-making. Within the framework of multi-criteria decision-making, the peculiarities of the problem that required decision-making need the incorporation of a broad collection of components and concepts. Listed below is a compilation of the most significant ones:

- Alternatives are "different possible courses of action" (different possible courses of conduct).
- "a measurable characteristic of an alternative" is how the attribute is described in the dictionary.
- "considering the performances of an alternative on the specific criteria for deciding on the alternative" is what is meant when we talk about "aggregation."

- We refer to decision variables as "components of alternatives' vector" in our formal definition.
- The term "feasible alternatives" is used to denote the decision space.
- The phrase "elements utilized to quantify an alternative to its attribute by assigning to the attribute numbers or symbols" is how the term "measures" is defined.
- Criteria are "tools for evaluating and comparing alternatives from the viewpoint of the consequences of their selection," according to one definition of the term.
- "how an alternative fulfills the need of a decision-maker regarding a given attribute" is the definition of "preferences."
- Choice, ranking, and sorting difficulties are all examples of the diverse kinds of challenges that might call for distinct approaches to decision making.

The solution to an Multi-criteria decision-making issue might be interpreted in a number of different ways. The procedure may be thought of as selecting the optimal (preferred) solution from among a group of alternatives. It is also possible to express it as the process of categorizing potential options (into various sets of preferences) before selecting a limited number of those options. In addition, the solutions to these challenges seek to specify the alternatives that are not dominant or that are effective. There is no way to transition from one non-dominated solution to another solution without giving up at least one of the criteria that this point can assist decision-makers choose a solution-set from among the set of non-dominated ones. This is because there is no way to shift from one non-dominated solution to another solution without giving up a non-dominated option. The following is a mathematical formulation of the definition of an Multi-criteria decision-making problem:

$$A = \{A_i \mid i = 1, 2, \dots, m\}$$

where A is a distinct and finite collection of alternatives, and m is the integer that denotes how many there are in total.

$$C = \{C_j \mid j = 1, 2, \dots, n\}$$

Multi-Criteria Decision-Making Categories

As was previously said, there is a plethora of Multi-criteria decision-making approaches, each of which has unique features that may be connected to a variety of factors ranging from the standard of the replies to the nature of the issue that is solved by these methods. In light of this, it is vital to determine the categorization of Multi-criteria decision-making problems in order to have a better knowledge of the Multi-criteria decision-making approaches, which in turn helps to pick a suitable strategy for the difficulties that are encountered. Numerous studies have identified several typologies and sub-groups, each of which takes into account a different facet of the situation at hand. Following a rundown of some

of the most important subgroups, this section moves on to provide an in-depth explanation of the classification method that is utilized most frequently.

A Process of Implicit Decision-Making in Learning Motivation

This term, "motivation," is used to characterize the many systems that are responsible for activating and directing behavior, and it is also used to refer to the processes themselves. However, despite the vast amount of research that has been done on the topic of educational motivation, it is still not totally clear how precisely learning behaviors get triggered or directed. This is despite the fact that there has been a substantial number of study undertaken on the issue. for example, pointed out that in recent years many writers have depicted motivation as "a complex interplay of individual processes and contextualized experiences".

TYPE 1 AND TYPE 2 FUZZY SETS

The concept of a Type 2 fuzzy set as an enlargement of the concept of a typical fuzzy set, which was referred to as a Type-1 fuzzy set. Type-1 fuzzy frameworks, the membership functions of which are Type-1 fuzzy sets, are unable to deal with vulnerabilities in a straightforward manner that are not straightforward.

There exist fuzzy frameworks of type 2, in which the membership functions that are either the precursor or the outcome are fuzzy sets of type 2. An example of such a set is a fuzzy set whose participation grades are themselves Type-1 fuzzy sets. These kinds of sets are very useful in situations when it is difficult to determine the appropriate membership function for a fuzzy set.

Fuzzy sets of type 1 are applied in situations when it is not possible to determine whether a component in a set belongs to the set as a 0 or a 1. This is also the case when the scenario is ambiguous to the extent that the difficulty situation, choosing the participation review even as a crisp number in the range of [0, 1], makes use of Type 2 fuzzy sets.

According to this, it is not necessary for us to have a significant amount of experience in fuzzy situations in order to make use of Type-2 fuzzy sets. There are there are certain genuine problems in which it is not possible to choose the appropriate kind of membership functions. for example, anticipating time series in light of the chaos that has been occurring in the information.

Another type of surveying is known as Type-1 fuzzy sets, which are believed to be a first-order estimate, and Type-2 fuzzy sets, which are thought to be a second-order estimate to the vulnerability in reality. Consequently, it is not inconceivable to take into consideration fuzzy arrangements of higher sorts, despite the fact that the unpredictability of the fuzzy framework swiftly increases.

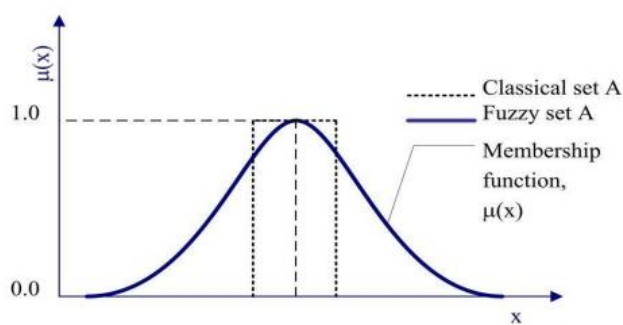
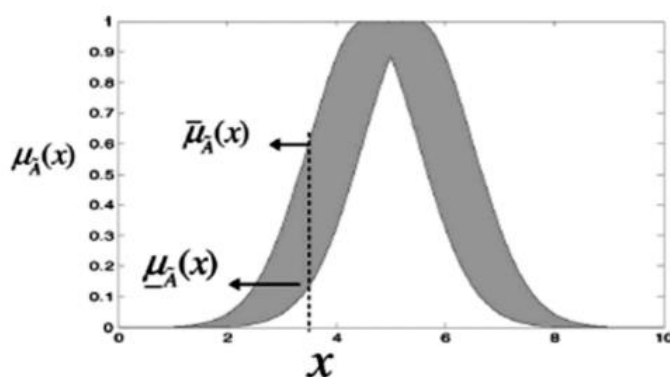


Figure 1: Type 1 Fuzzy Membership Function



Where, $x \in X$ Universe of Discourse and $\mu_A(x)$ membership function

Figure 2: Type 2 Fuzzy Set

NORMAL FUZZY NUMBERS

Triangular Fuzzy Number

A fuzzy number $\tilde{F} = ((\delta_1, \delta_2, \delta_3); \delta_i \in R)$ is known as a triangular fuzzy number, the membership function of which is defined as follows.

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x - \delta_1}{\delta_2 - \delta_1}; & \delta_1 \leq x < \delta_2 \\ 1; & x = \delta_2 \\ \frac{\delta_3 - x}{\delta_3 - \delta_2}; & \delta_2 \leq x < \delta_3 \\ 0; & \text{otherwise} \end{cases}$$

The α – cut of fuzzy number $(\delta_1, \delta_2, \delta_3)$ is an interval of confidence defined as

$$\tilde{F}_\alpha = [\delta_1^\alpha, \delta_3^\alpha] = [\delta_1 + (\delta_2 - \delta_1)\alpha, \delta_3 - (\delta_3 - \delta_2)\alpha]; \alpha \in (0,1]$$

A visual illustration of the alpha-cut is provided in the following:

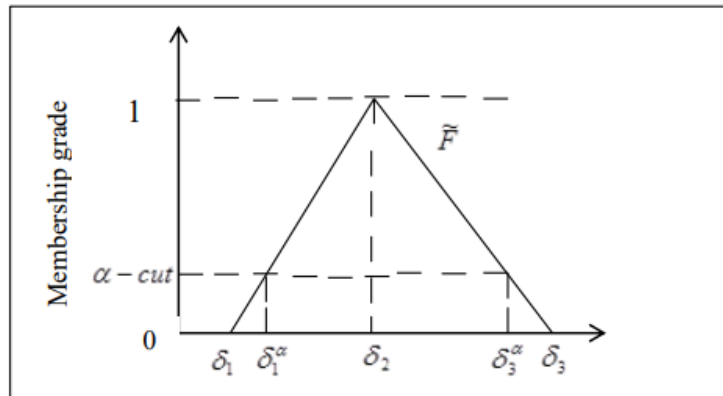


Figure 3: Triangular fuzzy number and alpha cut

Arithmetic interval analysis for triangular fuzzy numbers:

Let $\tilde{F}_1 = (\delta_1, \delta_2, \delta_3)$ and $\tilde{F}_2 = (\eta_1, \eta_2, \eta_3)$ exist in the form of interval numbers, consisting of two triangular fuzzy numbers having a lower limit and an upper limit on each side.

Define $* = \{+, -, \times, \div\}$ as arithmetic operation on $\tilde{F}_1$ and $\tilde{F}_2$.

In a representative manner, the mathematical operation that is given by

$$\begin{aligned} \tilde{F}_1 * \tilde{F}_2 &= [\delta_1, \delta_2, \delta_3] * [\eta_1, \eta_2, \eta_3] \\ &\bullet [\delta_1, \delta_2, \delta_3] + [\eta_1, \eta_2, \eta_3] = [\delta_1 + \eta_1, \delta_2 + \eta_2, \delta_3 + \eta_3] \\ &\bullet [\delta_1, \delta_2, \delta_3] - [\eta_1, \eta_2, \eta_3] = [\delta_1 - \eta_3, \delta_2 - \eta_2, \delta_3 - \eta_1] \\ &\bullet [\delta_1, \delta_2, \delta_3] \times [\eta_1, \eta_2, \eta_3] = [\delta_1 \times \eta_1, \delta_2 \times \eta_2, \delta_3 \times \eta_3] \\ &\bullet [\delta_1, \delta_2, \delta_3] \div [\eta_1, \eta_2, \eta_3] = [\delta_1, \delta_2] \times \left[\frac{1}{\eta_3}, \frac{1}{\eta_2}, \frac{1}{\eta_1} \right] = \left[\frac{\delta_1}{\eta_3}, \frac{\delta_2}{\eta_2}, \frac{\delta_3}{\eta_1} \right] \\ &\bullet k[\delta_1, \delta_2, \delta_3] = \begin{cases} [k\delta_1, k\delta_2, k\delta_3] & k > 0 \\ [k\delta_3, k\delta_2, k\delta_1] & k < 0 \end{cases} \end{aligned}$$

1.11.3 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{F} = ((\delta_1, \delta_2, \delta_3, \delta_4); \delta_i \in R)$ is called trapezoidal fuzzy number whose membership function is defined as

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x - \delta_1}{\delta_2 - \delta_1}; & \delta_1 \leq x < \delta_2 \\ 1; & \delta_2 \leq x < \delta_3 \\ \frac{\delta_4 - x}{\delta_4 - \delta_3}; & \delta_3 \leq x < \delta_4 \\ 0; & \text{otherwise} \end{cases}$$

The α – cut of fuzzy number $(\delta_1, \delta_2, \delta_3, \delta_4)$ is an interval of confidence defined as

$$\tilde{F}_\alpha = [\delta_1^\alpha, \delta_4^\alpha] = [\delta_1 + (\delta_2 - \delta_1)\alpha, \delta_4 - (\delta_4 - \delta_3)\alpha]; \alpha \in (0,1]$$

A visual illustration of the alpha-cut is provided in the following:

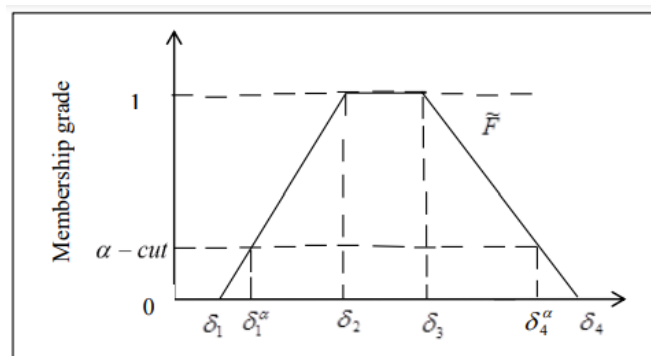


Figure 7: Trapezoidal fuzzy number and alpha cut

1.11.4 Arithmetic interval analysis for trapezoidal fuzzy numbers:

Let $\tilde{F}_1 = (\delta_1, \delta_2, \delta_3, \delta_4)$ $\tilde{F}_2 = (\eta_1, \eta_2, \eta_3, \eta_4)$ be two trapezoidal fuzzy numbers in the form of interval numbers with lower limit and upper limit.

Define $*$ = $\{+, -, \times, \div\}$ as arithmetic operation on $\tilde{F}_1$ and $\tilde{F}_2$.

Representatively, the arithmetic operation given by

$$\begin{aligned} \bullet \quad [\delta_1, \delta_2, \delta_3, \delta_4] \div [\eta_1, \eta_2, \eta_3, \eta_4] &= [\delta_1, \delta_2, \delta_3, \delta_4] \times \left[\frac{1}{\eta_4}, \frac{1}{\eta_3}, \frac{1}{\eta_2}, \frac{1}{\eta_1} \right] \\ &= \left[\frac{\delta_1}{\eta_4}, \frac{\delta_2}{\eta_3}, \frac{\delta_3}{\eta_2}, \frac{\delta_4}{\eta_1} \right] \end{aligned}$$

$$\bullet \quad k[\delta_1, \delta_2, \delta_3, \delta_4] = \begin{cases} [k\delta_1, k\delta_2, k\delta_3, k\delta_4] & k > 0 \\ [k\delta_4, k\delta_3, k\delta_2, k\delta_1] & k < 0 \end{cases}$$

$$\tilde{F}_1 * \tilde{F}_2 = [\delta_1, \delta_2, \delta_3, \delta_4] * [\eta_1, \eta_2, \eta_3, \eta_4]$$

$$\bullet \quad [\delta_1, \delta_2, \delta_3, \delta_4] + [\eta_1, \eta_2, \eta_3, \eta_4] = [\delta_1 + \eta_1, \delta_2 + \eta_2, \delta_3 + \eta_3, \delta_4 + \eta_4]$$

$$\bullet \quad [\delta_1, \delta_2, \delta_3, \delta_4] - [\eta_1, \eta_2, \eta_3, \eta_4] = [\delta_1 - \eta_4, \delta_2 - \eta_3, \delta_3 - \eta_2, \delta_4 - \eta_1]$$

$$\bullet \quad [\delta_1, \delta_2, \delta_3, \delta_4] \times [\eta_1, \eta_2, \eta_3, \eta_4] = [\delta_1 \times \eta_1, \delta_2 \times \eta_2, \delta_3 \times \eta_3, \delta_4 \times \eta_4]$$

1.11.5 Right Triangular Fuzzy Number

A fuzzy number $\tilde{F} = ((\delta_1, \delta_2); \delta_i \in R)$ is called right triangular fuzzy number if its membership function is defined as

$$\mu_{\tilde{F}}(x) = \begin{cases} 1; & x = \delta_1 \\ \frac{\delta_2 - x}{\delta_2 - \delta_1}; & \delta_1 < x \leq \delta_2 \\ 0; & \text{otherwise} \end{cases}$$

The α – cut of fuzzy number $[\delta_1, \delta_2]$ is an interval of confidence defined as

$$\tilde{F}_\alpha = [\delta_1^\alpha, \delta_2^\alpha] = [\delta_1, \delta_2 - (\delta_2 - \delta_1)\alpha]$$

The following image shows alpha-cut.

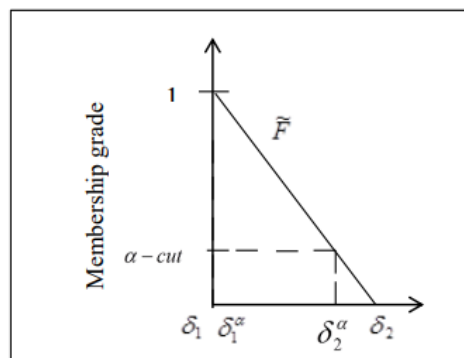


Figure 1.8: Right Triangular Fuzzy Number

1.11.6 Interval analysis in arithmetic:

Let $\tilde{F}_1 = (\delta_1, \delta_2)$ $\tilde{F}_2 = (\eta_1, \eta_2)$ be two right triangular fuzzy numbers in the form of interval numbers with lower limit and upper limit.

Define $*$ = $\{+, -, \times, \div\}$ as arithmetic operation on $\tilde{F}_1$ and $\tilde{F}_2$.

Representatively, the arithmetic operation given by

$$\begin{aligned} \tilde{F}_1 * \tilde{F}_2 &= [\delta_1, \delta_2] * [\eta_1, \eta_2] \\ &\bullet \quad [\delta_1, \delta_2] + [\eta_1, \eta_2] = [\delta_1 + \eta_1, \delta_2 + \eta_2] \\ &\bullet \quad [\delta_1, \delta_2] - [\eta_1, \eta_2] = [\delta_1 - \eta_2, \delta_2 - \eta_1] \\ &\bullet \quad [\delta_1, \delta_2] \times [\eta_1, \eta_2] = [\delta_1 \times \eta_1, \delta_2 \times \eta_2] \\ &\bullet \quad [\delta_1, \delta_2] \div [\eta_1, \eta_2] = [\delta_1, \delta_2] \times \left[\frac{1}{\eta_2}, \frac{1}{\eta_1} \right] = \left[\frac{\delta_1}{\eta_2}, \frac{\delta_2}{\eta_1} \right] \\ &\bullet \quad k[\delta_1, \delta_2] = \begin{cases} [k\delta_1, k\delta_2] & k > 0 \\ [k\delta_2, k\delta_1] & k < 0 \end{cases} \end{aligned}$$

ORIGIN OF FUZZY THEORY

It was first the fuzzy sets theory that was created. In the 1950s, he came to the realization that all problems that arise in the real world may be solved by using methods that are both analytical and efficient, in addition to making use of electronic devices. His conviction that standard techniques of system analysis were too particular for a significant number of the complex problems that are faced in real life was voiced in the year 1960. He was of the opinion that this was the case. He demonstrates that there is a need for a different kind of mathematics, which he calls fuzzy mathematics. He calls this unique type of mathematics. Within the context of probability distributions, there was no explanation offered for what it was. In the process of establishing this development, his goal was to make use of natural language in order to convey a feeling of uncertainty and imprecision in the facts and statements that were originally available. Fuzzy sets provide a mathematical method for expressing fuzziness, which may be found in the field of mathematics. The procedures of manufacture and development may take a considerable amount of time and money. This is due to the fact that engineering models and products need a high degree of precision. When it comes to technologies that are not straightforward, the cost is directly proportional to the degree of accuracy used. When a scientist or engineer is contemplating the application of fuzzy logic to a particular circumstance, it is essential for them to make advantage of the tolerance for imprecision that is included within the framework of fuzzy logic. Not only can a high level of accuracy result in costly charges, but it also results in a problem that is difficult to grasp and difficult to address.

It is possible to regard uncertainty as the total opposite of information when evaluated from an epistemological point of view. Incomplete, imprecise, fragmented, untrustworthy, ambiguous, conflicted, or deficient are all qualities that may be connected with information related with certain technological or scientific challenges. These traits may also be associated with information that is lacking. There is a

correlation between the amount of knowledge we have about a problem and the amount of uncertainty we have about the phrasing of the problem and the possible solutions. There is a lack of understanding about issues that are recognized to be unstable, difficult, or not adequately understood. These are the types of issues that are known to be problematic. When it comes to these matters, there is a significant amount of uncertainty that exists. It is possible for uncertainty to manifest itself in a variety of ways: it may be fuzzy (not pointed, inconsistent, unclear, or estimate), vague (not particular, amorphous), ambiguous (too many preferences, conflicting), or a form of arrogance; it may also be a form due to natural variability; or it may be uncertain in many respects due to natural variability (conflicting, random, chaotic, and unpredictable). In order to discover a suitable method for expressing uncertainty, it is vital for engineers to take into account the nature of the uncertainty that exists in a problem before making a decision. This is done in order to find a solution. Fuzzy sets are used in humanistic systems for the purpose of expressing ambiguity and uncertainty via the application of mathematical techniques.

Conclusion

This study presents an enhanced version of the non-normalized (generalized) Pentagonal Fuzzy Number, along with the features that it has. Additionally, the article discusses other aspects of the variant. Additionally, this work presents a number of operational properties of the Generalized Pentagonal Fuzzy Number. These features are described in it. The acquisition of the centroid of a Pentagonal Fuzzy Number has been determined and extended for the non-normalized form. This indicates that the centroid has been obtained. For the purpose of determining the median of a Pentagonal Fuzzy Number, a computation is performed. In addition to that, we introduced a novel species of fuzzy number that we referred to as Sequential Fuzzy Number. The analysis of real-world events might benefit from the use of this kind of fuzzy number. By taking into account the subjective judgments of experts, the Generalized Pentagonal Fuzzy Number has the potential to increase the efficacy of a decision-making system. Consequently, with the aid of Generalized Pentagonal Fuzzy number, we are able to capture imprecision or vagueness that is inherent in any decision-making system. This is because of the unique characteristics of fuzzy numbers.

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